that the circumferential variation is more a function of c/a than the absolute separation $2c$ or $2c/\lambda$. Also, comparing Tables I and II with Tables I and II from [1] shows that including both polarizations in the model reduces the circumferential variation of J_z.

In summary we conclude that, although small, the circumferential polarization of current has a significant effect on the circumferential variation of the longitudinal component of currents on a thin wire.

REFERENCES

[1] P. Tulyathan and E. H. Newman, "The circumferential variation of the axial component of current in closely spaced thin-wire antennas," *IEEE Trans. Antennas Propagat.*, vol. AP-27, no. 1, pp. 46–50, Jan. 1979.

[2] C. C. Kao, "Three-dimensional electromagnetic scattering from a circular tube of finite length," *J. Appl. Phys.*, vol. 40, no. 12, pp. 4732–4740, Nov. 1969.

[3] B. M. Duff, "Angular variation of current on moderately thin cylindrical scatterers," Univ. Mississippi Rep. AFOSR-TR-78-0769, Sept. 1976.

[4] E. H. Newman and D. M. Pozar, "Electromagnetic modeling of composite wire and surface geometries," *IEEE Trans. Antennas Propagat.*, vol. AP-26, no. 6, pp. 784–789, Nov. 1978.

[5] ——, "Considerations for efficient wire/surface modeling," *IEEE Trans. Antennas Propagat.*, vol. AP-28, no. 1, pp. 121–125, Jan. 1980.

[6] J. H. Richmond, "Radiation and scattering by thin-wire structures in the complex frequency domain," NASA Rep. NASA-CR-2396, July 1973.

Current Distribution on a Receiving Dipole Antenna

T. TANG AND M. W. GUNN

Abstract—**The current distribution on a receiving dipole antenna is dependent on its feed-point load impedance. Calculations based on the modified King–Middleton approximation have been made of the distribution for antenna half lengths of 0.1, 0.2, 0.25, and 0.3 λ. Representative values of the antenna load impedance, resistive, capacitive, and inductive, are used to show their effects on the antenna current distribution.**

I. INTRODUCTION

The current distribution along a linear receiving antenna feeding a conjugate matched load may be found by using the theorem of reciprocity and a knowledge of the current distribution of an identical transmitting antenna. If prior knowledge of the transmitting antenna current distribution is lacking, the current distribution of a receiving antenna with a matched load can be derived from the incident electric field [1]. Both approaches yield similar results.

Manuscript received August 21, 1980; revised January 15, 1981. This work was supported by the Radio Research Board Australia.

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However it is known that the current distribution along a receiving dipole antenna depends greatly on the load (Z_L). King [1] has given theoretical derivations for the conditions $Z_L = 0$, $Z_L = Z_0^*$, $|Z_L| \gg |Z_0|$, where Z_0 is the antenna feed-point impedance. These derivations confirmed experimental results reported by Morita [2].

The effect of an arbitrary load on the current distribution along a receiving dipole antenna is investigated in this communication. Under mismatched conditions, the current distribution can change so significantly that it no longer resembles that on a transmitting antenna. The simple sinusoidal approximation is then no longer valid.

II. METHOD OF APPROACH

It has been shown [3] that the quasi-zero-order (or King–Middleton) approximation of the current distribution along a receiving dipole antenna gives a relatively simple analytical solution and gives reasonably accurate results for dipole antennas less than two wavelengths long. Fig. 1 shows the schematic diagram of a receiving dipole antenna of half-length h, radius a, with a feed-point load impedance Z_L. According to the approximation, the current distrubition $I(z)$ may be written as

$$I(z) = V_{20}v(z) + U_{21}u(z)$$

where $v(z)$ is associated with a virtual voltage source V_{20} at the antenna feed point, and $u(z)$ is due to the charge distribution throughout the antenna element. U_{21} is a function of the incident electric field. The simplified version of this approach, quoting only the variables and equations that are required in the calculation, is described in the appendix.

III. RESULTS

Computed results for $h/\lambda = 0.10$, 0.20, 0.25, and 0.30 are shown in Figs. 2–5, respectively. In each case $h/a = 100$ and an incident electric field of 1 V/m are assumed. Resistive loads of 10 $k\Omega$, 1 $k\Omega$, and the extreme case of short circuit are used to obtain the results shown in Figs. 2(a), 3(a), 4(a), and 5(a). Results for capacitive loads of $R - jX$, $R - j10X$ and $R - j100X$, where R and X are, respectively, the resistance and the magnitude of the reactance of the antenna, are shown in Figs. 2(b), 3(b), 4(b), and 5(b). Figs. 2(c), 3(c), 4(c), and 5(c) exhibit the current distribution on an antenna with inductive loads of $R + jX$, $10\,R + jX$, and $100\,R + jX$. Inductive loads of $R + j2X$, $R + j5X$, and $R + j10X$ are used to obtain the graphs of Figs. 2(d), 3(d), 4(d), and 5(d).

As a check for numerical accuracies, the following results are obtained for the cases $h/\lambda = 0.25$, $a/\lambda = 0.007022$, $h/a = 35.60$.

	Computed	From [3, Table A2.1]
ψ_{dR}	6.21769	6.21771
Y_0, mS	$10.1630 - j4.4275$	$10.1704 - j4.4303$

In all cases, errors are less than 0.1 percent.

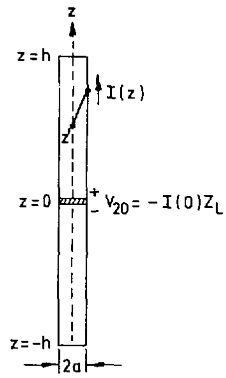

Fig. 1. Receiving dipole antenna.

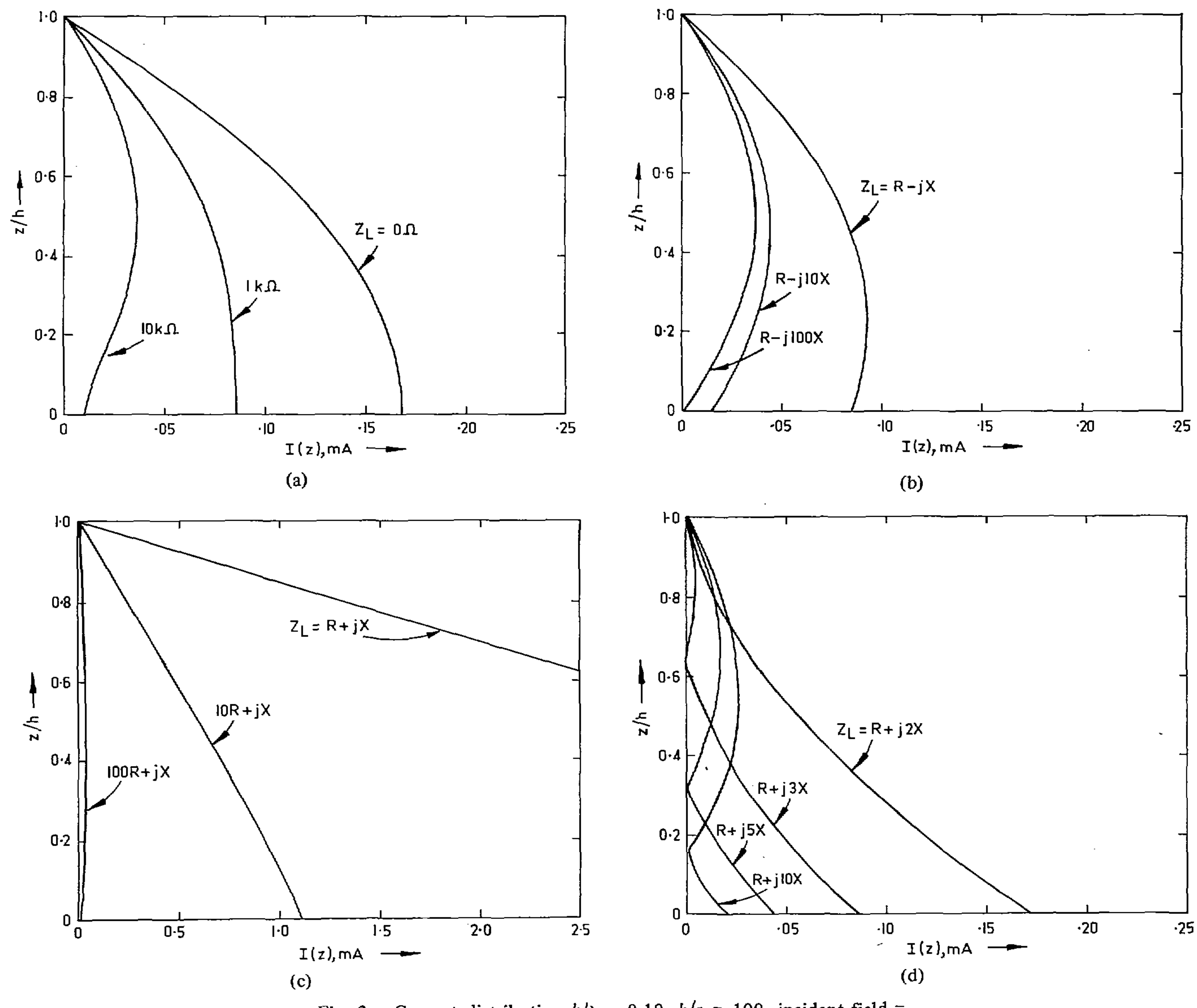

Fig. 2. Current distribution h/λ = 0.10, h/a = 100, incident field = 1 Vm^{-1}, and $Z_0 = R - jX$, R = 8.157 Ω, X = 589.4 Ω. (a) Resistive loads. (b) Capacitive loads. (c) Inductive loads (R increasing). (d) Inductive loads (X increasing).

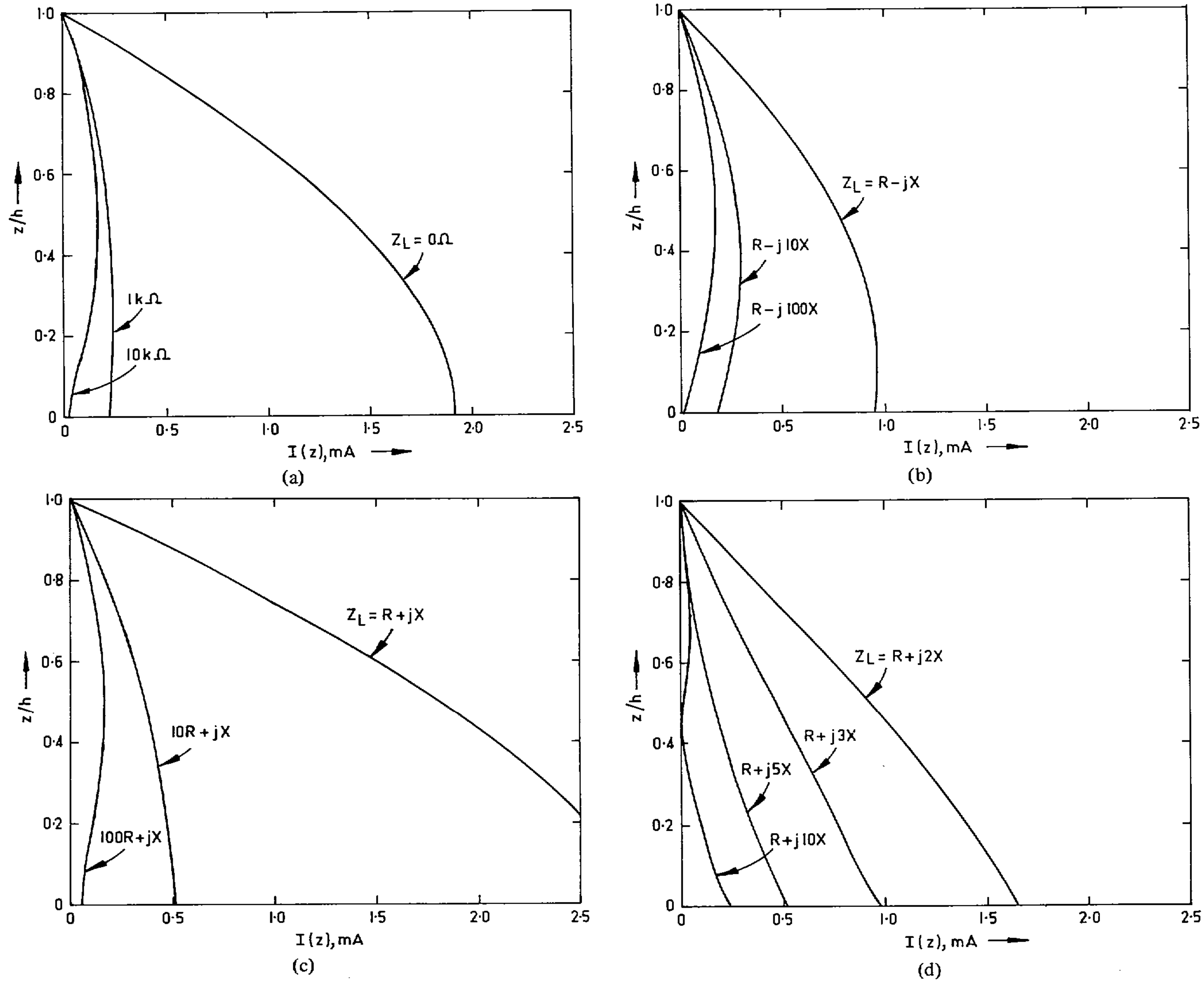

Fig. 3. Current distributions for $h/\lambda = 0.20$, $h/a = 100$, incident field = 1 Vm^{-1}, and $Z_0 = R - jX$, $R = 41.09\ \Omega$, $X = 115.9\ \Omega$. (a) Resistive loads. (b) Capacitive loads. (c) Inductive loads (R increasing). (d) Inductive loads (X increasing).

IV. DISCUSSION

From the results it is obvious that the current distribution along an antenna depends a great deal on the type of load connected to its feed points. With a resistive load, Figs. 2(a), 3(a), 4(a), and 5(a), the current distribution resembles that on a transmitting antenna when the load impedance is small compared to the magnitude of the antenna impedance for antennas with $h/\lambda < 0.25$. For longer antennas, $h/\lambda > 0.25$, the current distributions still have maximum values at the feed points. However when the load impedance is high, the dipole antenna behaves more like a two-element colinear scattering array. For this case, the current at the feed points decreases significantly, and the simple sinusoidal approximation for the current distribution is obviously no longer valid. It will be noted also that the longer the antenna length, the larger the peak value of the current distribution for an identical high-impedance load (e.g., 10 $k\Omega$).

For antennas with $h/\lambda < 0.25$ and capacitive loads, the current distribution follows the same trend as for resistive loads, as shown in Fig. 2(b) and Fig. 3(b). However for $h/\lambda > 0.25$, it is possible to resonate the antenna with a capacitive conjugate matched load. Under this condition the current distribution conforms to the simple sinusoidal approximation, as given by Fig. 4(b) and Fig. 5(b). With a larger capacitive load impedance, the current distribution again changes shape and its peak starts to move away from the feed-points.

For short antennas, an inductive conjugate matched load gives a simple sinusoidal current distribution as shown in Fig. 2(c) and Fig. 3(c). When $h/\lambda \geqslant 0.25$, an inductive load will not produce resonance and its effect on the current distribution follows the same trend as that of a resistive load, Fig. 4(c) and Fig. 5(c).

Fig. 2(d)–Fig. 5(d) show the adverse effect on the current distribution caused by a highly inductive load. It is obvious from the graphs that small "loops" start to appear as the load

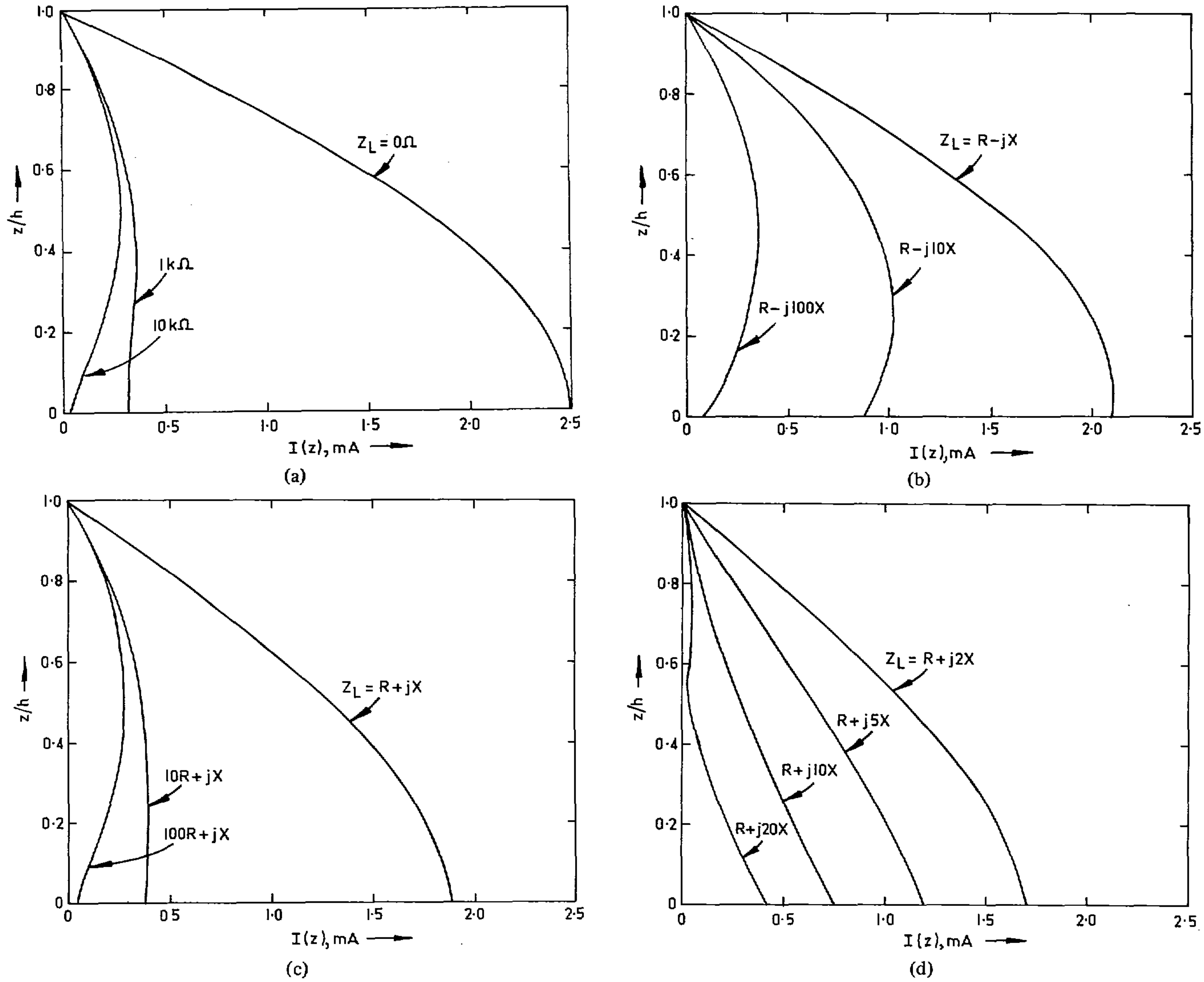

Fig. 4. Current distributions for $h/\lambda = 0.25$, $h/a = 100$, incident field = 1 Vm^{-1}, and $Z_0 = R + jX$, $R = 80.69\ \Omega$, $X = 39.12\ \Omega$. (a) Resistive loads. (b) Capacitive loads. (c) Inductive loads (R increasing). (d) Inductive loads (X increasing).

inductive reactance is increased beyond a certain value. This "threshold value" of the inductive reactance, however, varies for different h/λ ratio. The "loops" get larger as the reactance in increased further. In the limit, the current distribution assumes the shape corresponding to the load impedance approaching infinity (open circuit).

V. CONCLUSION

The King–Middleton approximation has been used to calculate the current distribution on a receiving dipole antenna with an arbitrary load. Distributions have been determined for antenna half lengths of 0.1, 0.2, 0.25, 0.3λ for load impedances in the range $Z_L = 0$ to $Z_L = \infty$.

It has been shown that the current distribution changes shape as the feed-point load impedance is varied. For large resistive loads, the maximum of the current distribution moves toward the middle point of the antenna element. A similar behavior is observed for capacitive loads. For inductive loads a current minimum appears on the antenna element and moves towards the feedpoint as the inductive reactance increases.

It is concluded that the simple sinusoidal current distribution approximation is valid only when the load impedance is small or is equal to the complex conjugate of the antenna feed-point impedance.

APPENDIX

If it is assumed that the transmitting and receiving antennas are parallel and nonstaggered, the current distribution on the receiving antenna $I(z)$ due to the incident electric field from the transmitting antenna can be found approximately from the integral equation below [3]:

$$\int_{-h}^{h} I(z^1) K_d(z, z^1)\, dz^1$$

$$= \frac{j4\pi}{\zeta_0 \cos k_0 h} \left[\tfrac{1}{2} V_{20} \sin k_0(h - |z|) + (U + U_{21})(\cos k_0 z - \cos k_0 h)\right] \tag{1}$$

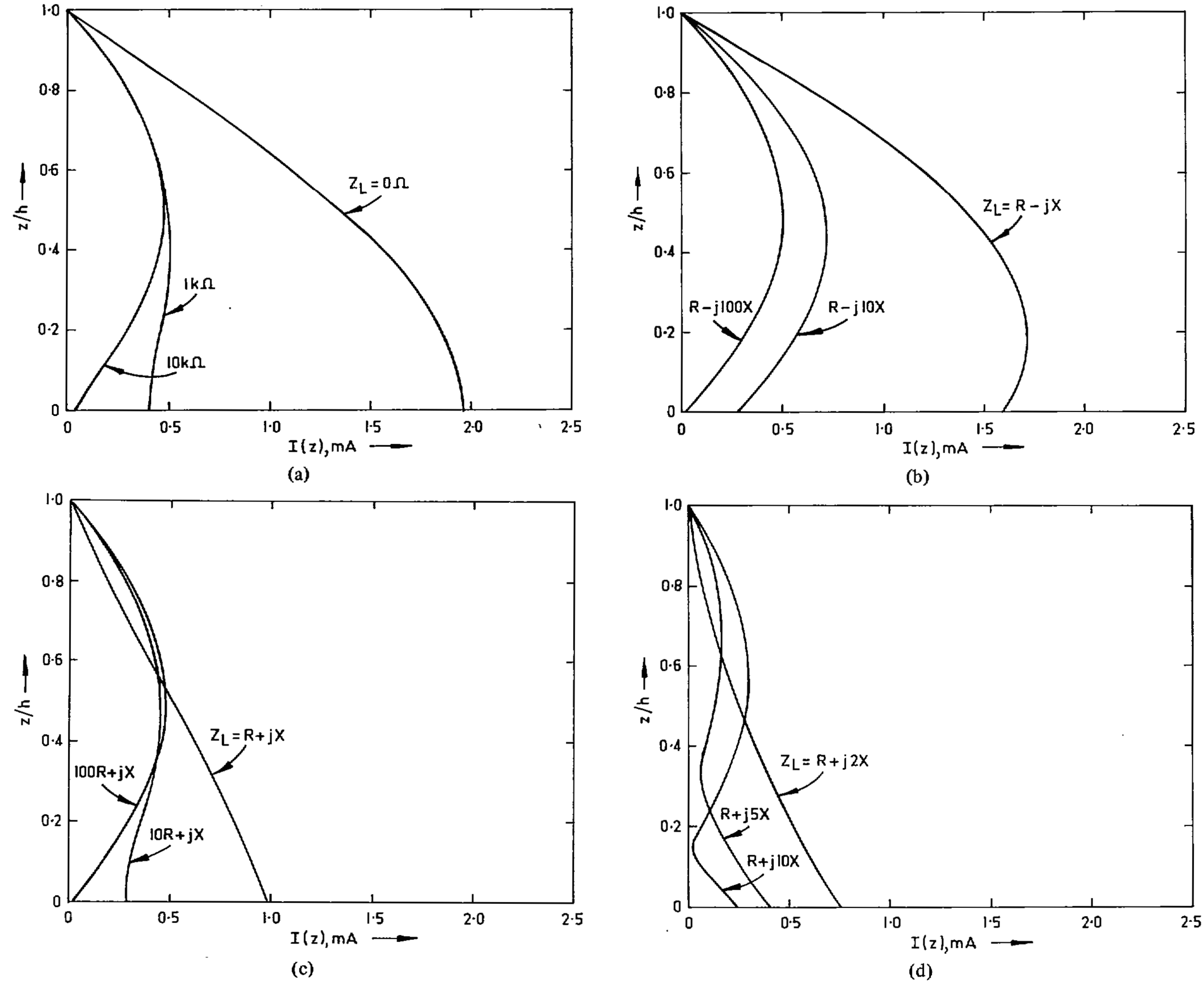

Fig. 5. Current distributions for $h/\lambda = 0.30$, $h/a = 100$, incident field = 1 Vm^{-1}, and $Z_0 = R + jX$, $R = 147.0\ \Omega$, $X = 189.3\ \Omega$. (a) Resistive loads. (b) Capacitive loads. (c) Inductive loads (R increasing). (d) Inductive loads (X increasing).

where

$$
\begin{aligned}
K_d(z, z^1) &= K_{22}(z, z^1) - K_{22}(h, z^1)\\
K_{22}(z, z^1) &= [\exp(-jk_0 R_{22})]/R_{22}\\
K_{22}(h, z^1) &= [\exp(-jk_0 R_{2h})]/R_{2h}\\
R_{22} &= \sqrt{(z - z^1)^2 + a^2}\\
R_{2h} &= \sqrt{(h - z^1)^2 + a^2}\\
\zeta_0 &= \omega\mu_0/k_0 = 120\pi\\
V_{20} &= -I(0)Z_L.
\end{aligned}
$$

Z_L is the load impedance connected across antenna feed point, U depends on V_{20}, and U_{21} depends on the incident field. A solution to (1) may be written as

$$I(z) = V_{20}v(z) + U_{21}u(z) \tag{2}$$

where for $k_0 h < \pi/2$,

$$
\begin{aligned}
v(z) = \frac{j2\pi}{\zeta_0 \psi_{dR} \cos k_0 h}[&\sin k_0(h - |z|)\\
&+ T(h)(\cos k_0 z - \cos k_0 h)]
\end{aligned}
$$

$$u(z) = \frac{j4\pi}{\zeta_0[\psi_{dU} \cos k_0 h - \psi_U(h)]}(\cos k_0 z - \cos k_0 h)$$

with

$$
\begin{aligned}
T(h) &= -\left[\frac{\psi_V(h) - j\psi_{dI}F_0(h)}{\psi_U(h) - \psi_{dU}F_0(h)}\right]\\
\psi_V(h) &= Ca(h, h)G_0(h) - S_a(h, h)F_0(h)\\
\psi_U(h) &= Ca(h, h) - E_a(h, h)F_0(h)
\end{aligned}
$$

$$\psi_{dU} = \frac{[C_a(h,0) - C_a(h,h)] - [E_a(h,0) - E_a(h,h)]\,F_0(h)}{1 - F_0(h)}$$

$$\psi_{dI} = \mathrm{Im}\left(\frac{[C_a(h,0) - C_a(h,h)]\,G_0(h) - [S_a(h,0) - S_a(h,h)]\,F_0(h)}{1 - F_0(h)}\right)$$

$$\psi_{dR} = \mathrm{Re}\,\frac{1}{G_0(h)}\,\{[C_a(h,0) - C_a(h,h)]\,G_0(h) - [S_a(h,0) - S_a(h,h)]\,F_0(h)\}$$

$$C_a(h,z) = \int_0^h \cos k_0 z^1 \left(\frac{e^{-jk_0R_1}}{R_1} + \frac{e^{-jk_0R_2}}{R_2}\right) dz^1$$

$$S_a(h,z) = \int_0^h \sin k_0 z^1 \left(\frac{e^{-jk_0R_1}}{R_1} + \frac{e^{-jk_0R_2}}{R_2}\right) dz^1$$

$$E_a(h,z) = \int_0^h \left(\frac{e^{-jk_0R_1}}{R_1} + \frac{e^{-jk_0R_2}}{R_2}\right) dz^1$$

$$R_1 = \sqrt{(z - z^1)^2 + a^2}$$

$$R_2 = \sqrt{(z + z^1)^2 + a^2}$$

$$G_0(h) = \sin k_0 h$$

$$F_0(h) = \cos k_0 h.$$

For $k_0 h \geqslant \pi/2$, we use

$$v^1(z) = \frac{-j2\pi}{\zeta_0 \psi^1_{dR}}\,[(\sin k_0|z| - \sin k_0 h) + T^1(h)(\cos k_0 z - \cos k_0 h)]$$

$$\psi^1_{dR} = \mathrm{Re}\left\{\left[C_a\left(h, h - \frac{\lambda}{4}\right) - C_a(h,h)\right] G_0(h) - \left[S_a\left(h, h - \frac{\lambda}{4}\right) - S_a(h,h)\right] F_0(h)\right\}$$

$$T^1(h) = -\frac{T(h) + \sin k_0 h}{\cos k_0 h}, \qquad h \neq \frac{\lambda}{4}$$

$$= \frac{\psi_{dU} - j\psi_{dI} - S_a\left(\frac{\lambda}{4}, \frac{\lambda}{4}\right) + E_a\left(\frac{\lambda}{4}, \frac{\lambda}{4}\right)}{C_a\left(\frac{\lambda}{4}, \frac{\lambda}{4}\right)}, \qquad h = \frac{\lambda}{4}.$$

It can then be shown that

$$V_{20} = -U_{21} u(0) \frac{Z_0 Z_L}{Z_0 + Z_L} \tag{3}$$

where $Z_0 = 1/v(0)$. By combining (2) and (3), we obtain

$$I(z) = U_{21}\left[u(z) - \frac{Z_0 Z_L}{Z_0 + Z_L}\, u(0) v(z)\right]. \tag{4}$$

REFERENCES

[1] R. W. P. King, *The Theory of Linear Antennas*. Cambridge, MA: Harvard Univ., 1956, Ch. IV.

[2] T. Morita, "The measurements of current and charge distributions on cylindrical antennas," *Proc. IRE*, vol. 38, pp. 898–904, Aug. 1950.

[3] R. W. P. King and C. W. Harrison, Jr., *Antennas and Waves: A Modern Approach*. Cambridge, MA: MIT, 1969, Ch. 8.

Dielectric Rod Leaky-Wave Antennas for Millimeter-Wave Applications

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Abstract—**Dielectric rod leaky-wave antennas have the property of being frequency scannable. Bounds are derived for the proper range of spacing of perturbations along the rod to avoid an intrusion upon a grating lobe when frequency scanning. In addition some experimental results are reported on sidelobe levels and polarizations in the far field for these antennas made from a material with $\epsilon_r = 2.33$ at 81.5 GHz.**

INTRODUCTION

Dielectric rod antennas with rectangular cross sections have been the focus of attention as a low-cost alternative for use with millimeter-wave integrated circuits. One type of dielectric rod antenna is a linear array whose radiation elements are excited by a surface wave propagating along the dielectric rod. We call this type of antenna a leaky-wave antenna. Klohn *et al.* [1] reported on the frequency scanning properties and some design criteria for this type of antenna. This communication attempts to augment this information by a study of the proper range of spacing of perturbations, sidelobe levels, and other characteristics. Examples of this type of antenna and the coordinate system used in this paper are shown in Fig. 1.

PROPER RANGE OF SPACING OF PERTURBATIONS

If one considers the leaky-wave antenna as a linear array, the phase difference between the adjacent elements of the

Manuscript received August 28, 1980; revised December 8, 1980.

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